Evaporation of charged bosonic condensate in cosmology

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Abstract

Cosmological evolution of equilibrium plasma with a condensate of U(1)-charged bosonic field is considered. It is shown that the evaporation of the condensate is very much different from naive expectations, discussed in the literature, as well as from evaporation of non-equilibrium neutral condensate. The charged condensate evaporates much slower than the decay of the corresponding bosons. The evaporation rate is close to that of the cosmological expansion. The plasma temperature, in contrast, drops much faster than usually, namely as the third power of the cosmological scale factor. As a result the universe becomes very cold and the cosmological charge asymmetry reaches a huge value.

1 Introduction

Bosonic condensates probably existed in the early universe and played an important role in the cosmological history. Well known examples are the classical real inflaton field, Φ [1], or complex field, χ, describing supersymmetric bosonic condensate, which carries baryonic, leptonic, or some other $U(1)$-charge [2]. Evaporation of the inflaton produced particles creating primeval plasma, while evaporation of χ could generate baryon or lepton asymmetry of the universe. Though evaporations of a real field condensate and a complex one share some similarities, the rates of the processes are very much different. In the case of the inflaton the rate of evaporation is determined by the particle production rate and may be quite large, while the evaporation of a charged condensate with a large charge asymmetry is much slower and is determined by the universe expansion rate $H = \dot{a}/a$ during most of its history. The impact of a large charge asymmetry on the process of evaporation has been considered in ref. [3] (see also [4]). A proper account of thermal equilibrium with a large charge asymmetry strongly changes results of refs. [5]-[9], where evaporation of bosonic condensate was considered. These works are applicable to the case of evaporation of uncharged condensate (e.g. inflaton) but evaporation of a charged condensate proceeds in a much different way, determined by thermal equilibrium with a large chemical potential and not just by the rate of particle production into plasma.

Here we elaborate the approach of ref. [3] and present more accurate and detailed calculations. It is found in particular that the plasma temperature in the presence of condensate drops very fast, as $T \sim 1/a^3$, where $a(t)$ is the cosmological scale factor. After the condensate disappears the cooling returns to the standard relativistic law, $T \sim 1/a$. We calculated the magnitude of the cosmological charge asymmetry produced as a result of the evaporation of the condensate and found that it might be much larger than unity because of the fast cooling of the plasma.

2   Pre-equilibrium evolution

We consider a charged classical scalar field χ at the final stage of its evolution, when its amplitude is small enough and the potential is dominated by the quadratic term

$$U(\chi) = m^2 |\chi|^2, \tag{1}$$

while quartic or higher terms can be neglected. The equation of motion for homogeneous χ has the form:

$$\ddot{\chi} + 3H\dot{\chi} + m^2\chi = 0, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter and $a(t)$ is the cosmological scale factor.

When the mass m of χ becomes larger than H the solution to equation (2) becomes oscillating:

$$\chi(t) = \chi_1 \cos mt + \chi_2 \sin mt \tag{3}$$

where the coefficients $\chi_{1,2}$ are slowly varying functions of time. In particular, at the matter dominated (MD) regime they behave as $\chi_{1,2} \sim a^{-3/2}$. Such a regime would be realized if cosmic energy density is dominated by the field χ itself. The energy density of the latter is equal to

$$\rho_\chi = m^2 \left(|\chi_1|^2 + |\chi_1|^2 \right) \tag{4}$$

The charge density of χ is expressed through the functions $\chi_{1,2}$ as:

$$Q_\chi = m \left(|\chi_1|^2 - |\chi_1|^2 \right) \tag{5}$$

It is clear that χ_1 and χ_2 correspond respectively to the contribution of particles and antiparticles. For what follows we will introduce a dimensionless parameter κ which describes the relative magnitude of the charge density:

$$\kappa = \frac{mQ_\chi}{\rho_\chi} = \frac{|\chi_1|^2 - |\chi_1|^2}{|\chi_1|^2 + |\chi_1|^2} \tag{6}$$

Evidently, at this stage $|\kappa| \leq 1$.

The oscillating field χ starts to produce lighter particles to which it is coupled (in many cases this can be understood as a decay of massive χ-bosons at rest into light particles).

This process is characterized by the decay width Γ. Here we will not consider the kinetics of the decay and do not need to specify the magnitude of Γ. It is assumed usually, but often *incorrectly*, that the condensate decays with the rate Γ and to the moment when $H < \Gamma$ it practically disappears transforming its energy and charge into those of light particles. The evolution of the energy densities of the condensate and relativistic plasma in this case are described by the equations:

$$\dot{\rho}_c = -3H\rho_c - \Gamma\rho_c$$
$$\dot{\rho}_r = -4H\rho_r + \Gamma\rho_c \tag{7}$$

and indeed ρ_c seems to disappear when the time is large enough, $t \sim 1/H > 1/\Gamma$. However, in the case when the charge density of the condensate is large, $\kappa \sim 1$, a large chemical potential prevents from fast evaporation of a charged condensate [3]. To describe this phenomenon properly one needs to include the contribution of inverse reactions into eq. (7). The problem is rather complicated and we postpone it for the future. However, one can still use eqs. (7) at the initial stage of the process when the amount of relativistic particles is small and inverse reactions can be neglected.

Here we consider simpler but practically interesting case when a partial decay of the condensate creates thermally equilibrium plasma which cools down by the universe expansion. We will show that the evolution of such plasma and, in particular, the decay rate of the condensate is determined by the cosmological expansion rate, H, which is much slower than the decay rate Γ. The rate of the decay of a charged scalar field into vacuum was considered in ref. [10]. Parameter κ introduced above (6) is convenient for the description of such equilibrium system. However, the value of κ at the equilibrium stage should be larger than its initial value. Indeed the density of a conserved charge drops down as $1/a^3$, while the energy density of mixed non-relativistic and relativistic matter drops somewhat faster. As a result κ rises and can even become larger than 1. To estimate its variation we use eqs. (7). They can be analytically solved giving:

$$\frac{\kappa}{\kappa_{in}} = \left[e^{-\eta} + \int_0^\eta d\eta' e^{-\eta'} \frac{a(\eta' + \eta_{in})}{a(\eta' + \eta_{in})} \right]^{-1}, \tag{8}$$

where $\eta = \Gamma(t - t_{in})$ and $\eta_{in} = \Gamma_{in}t$. If the universe expansion is dominated by the non-relativistic condensate, the scale factor behaves as $a(t) = (t/t_{in})^{2/3}$ and the integral can be easily evaluated. For small η: $\kappa/\kappa_{in} \approx 1 + 0.4\eta$, while asymptotically $\kappa/\kappa_{in} \sim a$, the latter is evident. However, as we have already mentioned above, the regime (8) remains true only till the inverse reactions can be neglected i.e. till equilibrium is established.

Now we will show that for a large magnitude of the condensed field, when the particle number density in the condensate, $n_c = C(t)m^3$ is large, i.e. $C \gg 1$, thermal equilibrium is quickly established when $\eta \ll 1$ and thus at that moment $|\kappa|$ cannot noticeably exceed unity. We assume that initially the cosmological matter was dominated by the condensate of the field χ and the contribution from light particles (quarks, leptons and photons) can be neglected. The kinetic equation governing the production of light particles reads

$$\dot{n}_0 + 3Hn_0 = \Gamma C m^3 \tag{9}$$

where n_0 is the number density of the produced massless particles. The number density of χ-particles in the condensate initially drops as $C(t) = C_{in}/a^3$ and the solution to equation (9) is easily found:

$$n_0(t) = \Gamma m^3 C_{in} \frac{t - t_{in}}{a^3(t)} \tag{10}$$

Thermal equilibrium would be established when $n_0(t)$ becomes of the order of its equilibrium magnitude, $n_0(t) \geq n_0^{(eq)}$. To this end the following condition is necessary:

$$\frac{2}{3} \sqrt{\frac{3C}{8\pi}} \frac{m^3}{n_0^{(eq)}} \frac{\Gamma m_{Pl}}{m^2} > 4, \tag{11}$$

which is evidently satisfied for a large region of the parameter values. The mass of χ-field is, of course, much smaller than the Planck mass. It normally can be in the range $10^3 - 10^{10}$ GeV. The time during which the equilibrium is established is quite short,

$$\Gamma(t_{eq} - t_0) = \frac{n_0^{(eq)}}{m^3 C} \ll 1. \tag{12}$$

On more comment may be in order here. From kinetic equations it follows that the number density of relativistic particles and χ-particles should decrease as $1/a^3$, if the system is very close to equilibrium. However, as we see in what follows, the number densities of quarks or leptons remains constant during rather large period, because they are proportional to cube of their chemical potential, eq. (24), which remains constant for large asymmetry, $\mu = m$, where μ and m are respectively chemical potential and the mass of χ (see the following section). Similar behavior is true for their energy densities, which naively should drop down but remain constant, see eq. (22). This kind of evolution is created by deviations from equilibrium which are proportional to H/Γ and small, but they give corrections of the order of unity to the collision integrals because of a large magnitude of the latter.

3   Thermal equilibrium

We consider the case when χ decays into the following channel:

$$\chi \rightarrow 3q + l \tag{13}$$

where q and l are quarks and leptons respectively. Analogous process exists for antiparticles. Leptonic, Q_L, and baryonic, Q_B, charges are assumed to be conserved in this decay. Thus the accumulated charge of the condensate is transformed into charges of light particles. This is the essence of baryogenesis scenario of ref. [2].

Particle distribution functions in thermal equilibrium are described by two parameters, their common temperature T and chemical potentials μ_j, which could change with time. As is well known, the equilibrium distribution of fermions is:

$$f_f = \frac{1}{\exp\left(\frac{E_f - \mu_f}{T}\right) + 1} \tag{14}$$

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and that of bosons is

$$f_b = \frac{1}{\exp\left(\frac{E_b - \mu_b}{T}\right) - 1} \tag{15}$$

where E are energies of the corresponding particles. Fermionic chemical potentials may have arbitrary values, while chemical potential of bosons is restricted from above by the value of the boson mass, $\mu_b < m$, otherwise the distribution function f_b would not be positive. If charge asymmetry of bosons is so big that even with maximum allowed $\mu_b = m$ such asymmetry could not be realized, then bosonic condensate must be formed and the equilibrium distribution becomes:

$$f_b = (2\pi)^3 m^3 C \delta(\mathbf{p}) + \frac{1}{\exp\left(\frac{E_b - m}{T}\right) - 1} \tag{16}$$

where $\mathbf{p}$ is the three-momentum of the bosons. Thus in the case of a large charge asymmetry the equilibrium plasma is still described by two parameters/functions, $T(t)$ and $C(t)$, the latter being the number density of particles in the condensate in units m^3.

The magnitude of chemical potentials of massless fermions is determined by the conditions of thermal equilibrium and in the particular case under consideration they are:

$$\mu_q = \mu_l = \mu_\chi / 4 \tag{17}$$

Here we have used the equilibrium condition

$$\sum_{in} \mu_{in} = \sum_{fin} \mu_{fin}, \tag{18}$$

where the summation is made over all particles in initial ("in") and final ("fin") states participating in all relevant reactions. For the system with two conserved charges, Q_B and Q_L in the case under consideration, there should be two independent chemical potentials. We excluded one of them imposing the condition $Q_B - Q_L = 0$.

The energy and, say, lepton charge density of particle species j are given respectively by:

$$\rho_j = \frac{1}{2\pi^2} \int dp p^2 E \left[f_j(E) + \bar{f}_j(E) \right] \tag{19}$$

$$p_j = \frac{1}{6\pi^2} \int dp p^2 \frac{p^2}{E} \left[f_j(E) + \bar{f}_j(E) \right] \tag{20}$$

$$L_j = \frac{Q_{Lj}}{2\pi^2} \int dp p^2 E \left[f_j(E) - \bar{f}_j(E) \right] \tag{21}$$

where $\bar{f}$ is the distribution function for antiparticles and L_j is the leptonic charge of particle j. In equilibrium chemical potentials of antiparticles are opposite to those of particles, $\bar{\mu} = -\mu$. We assume that $Q_{L_l} = Q_{L_\chi} = 1$. If chemical potential of χ is positive and equal to m, then $C \neq 0$ for χ, while $\bar{C} = 0$ for anti-χ, and vice versa if $\mu_\chi = -m$. Integration over angles in eqs. (19-21) is trivially performed because the distribution functions in homogeneous and isotropic universe are also isotropic.

For massless quarks and leptons the integrals can be taken analytically. The total energy density of all relativistic fermions plus photons is equal to:

$$\rho_{rel} = \rho_f + \rho_\gamma = N_f T^4 \left(\frac{7\pi^2}{120} + \frac{\xi_f^2}{4} + \frac{\xi_f^4}{8\pi^2} \right) + \frac{\pi^2}{15} T^4 = m^4 \left(\gamma_1 + \gamma_2 \xi^{-2} + \gamma_3 \xi^{-4} \right), \quad (22)$$

where $\xi_f = \mu_f/T = \mu_\chi/(4T)$ is dimensionless chemical potential of fermions, $\xi = \mu_\chi/T$ is the chemical potential of χ, and

$$\gamma_1 = \frac{N_f}{2^{11}\pi^2}, \quad \gamma_2 = \frac{N_f}{2^6}, \quad \gamma_3 = \frac{\pi^2}{15} \left(\frac{7N_f}{8} + 1 \right). \quad (23)$$

Here N_f is the total number of massless fermion species, quarks and leptons included. For three quark-lepton families, three quark colors, and two spin states: $N_f = 48$ (the contribution of antiparticles is already included inside the brackets of eq. (22)).

The charge density of massless leptons is:

$$L = \frac{N_l T^3 \xi_l}{6} \left(1 + \frac{\xi_l^2}{\pi^2} \right) = m^3 \left(\beta_0 + \beta_1 \xi^{-2} \right) \quad (24)$$

where $\xi_l = \mu_\chi/4T$, $N_l = 12$ is the number of lepton species which include 3 families of particles with two spin states, and

$$\beta_0 = \frac{N_l}{6 \cdot 2^6 \pi^2}, \quad \beta_1 = \frac{N_l}{24}. \quad (25)$$

The subtle points are the number of right-handed neutrino states and the contribution of heavy t-quarks which may be absent at relatively small T created after χ-evaporation. In particular, if the number of neutrino species is twice smaller due to an absence of ν_R, then N_l would be 9, and $N_f = 45$; if instead we consider 6 neutrinos but exclude the top quark we would find $N_l = 12$ and $N_f = 42$. In both these cases it can be easily seen that the equilibrium of the reaction $\chi \to 3q + l$, together with the condition $Q_B = Q_L$, no longer leads us to the simple relation $\mu_q = \mu_l = \mu_\chi/4$. Indeed we find different values of chemical potentials of quarks and leptons in terms of that of the χ particles. We have done calculations for these two different scenarios, with the calculated values for μ_q and μ_l, and we have found that the picture is unchanged, and all the results presented below are practically the same for different particle content.

The integrals which determine the energy and charge density of massive χ-field cannot be taken analytically but in the limit of low temperature, $T \ll m$, their approximate expressions are easily found. In what follows we express everything in units of χ-mass, m. In particular, when condensate is non-vanishing, i.e. $\mu = m$ and $C \neq 0$ we obtain:

$$m^{-4} \rho_\chi \approx C + \frac{T^4 \xi^{5/2}}{\sqrt{2}\pi^2 m^4} \int \frac{dy \sqrt{y}}{e^y - 1} \left(1 + \frac{9}{4} \frac{y}{\xi} \right) = C + \alpha_1 \xi^{-3/2} + \frac{9}{5} \alpha_2 \xi^{-5/2}, \quad (26)$$

$$m^{-4} p_\chi \approx \frac{\sqrt{2} T^4 \xi^{3/2}}{3\pi^2 m^4} \int \frac{dy y^{3/2}}{e^y - 1} = \alpha_3 \xi^{-5/2}, \quad (27)$$

$$m^{-3} Q_\chi \approx C + \frac{T^3 \xi^{3/2}}{\sqrt{2}\pi^2 m^3} \int \frac{dy \sqrt{y}}{e^y - 1} \left(1 + \frac{5}{4} \frac{y}{\xi} \right) = C + \alpha_1 \xi^{-3/2} + \alpha_2 \xi^{-5/2}, \quad (28)$$

where $\alpha_1 = 0.164$, $\alpha_2 = 0.16$, and $\alpha_3 = 8\alpha_2/15$. Notice that the condensate creates zero pressure because of $\delta(\mathbf{p})$ in the distribution function (16).

4 Condensate decay and plasma temperature

Since by assumption the total leptonic and baryonic charges were conserved when χ relaxed down to zero in $U(1)$-symmetric quadratic potential (1) and decayed into channel (13), each of them satisfied the conservation equation:

$$\dot{Q}_{tot} + 3HQ_{tot} = 0. \tag{29}$$

Thus leptonic or baryonic charge density evolved as:

$$Q_{tot} \sim Q_{in}/a^3 \tag{30}$$

We have two unknown functions $T(a)$ and $C(a)$ if condensate is present or $T(a)$ and $\mu(a)$ after condensate evaporation. To determine the law of their evolution we need another equation which is the law of covariant conservation of the total energy-momentum tensor. In Friedman-Robertson-Walker (FRW) metric this conservation law has the form:

$$\dot{\rho}_{tot} = -3H \left(\rho_{tot} + p_{tot} \right), \tag{31}$$

where ρ_{tot} includes the energy density of relativistic particles (photons and fermions), and the energy density of χ, either relativistic or not, depending upon the initial conditions. The pressure of relativistic matter is $p_{rel} = \rho_{rel}/3$ and the pressure of χ is given by eq. (20) which in non-relativistic case turns into (27).

It is convenient to look for the evolution of C and T as functions of the scale factor a, taking the latter as the independent variable. In this case the Hubble parameter disappears from the equation and we obtain:

$$\rho'_{tot} = -3 \left(\rho_{tot} + p_{tot} \right)/a, \tag{32}$$

where prime means derivative with respect to the scale factor a.

The initial values of $\xi = m/T$ and C can be expressed through the initial values of the total energy density, $\rho^{(in)}$ and, say, leptonic charge density, $Q_l^{(in)} = \kappa \rho^{(in)}$. As we have already mentioned $|\kappa| \leq 1$ or it may be slightly larger than 1. The values of $C^{(in)}$ and $T^{(in)}$ are presented in figures 1 to 5 as functions of $\rho^{(in)}$ and κ. In particular, figs. 1 to 3 show how T_{in} and C_{in} depend upon κ when ρ_{in} is fixed, while figs. 4 and 5 deal with their dependence upon the total ρ_{in}, with fixed κ. If $\kappa \approx 1$ the initial temperature is low and $\xi^{(in)} \gg 1$. In particular, for $\kappa = 1$ the initial temperature does not depend upon the initial value of ρ and is equal to:

$$T_{in} \approx 0.05m \tag{33}$$

However, even if κ is slightly smaller than unity, $T^{(in)}$ becomes much higher and, for large ρ_{in}, even relativistic initial state could be realized, i.e. $m/T < 1$. E.g. for $\kappa = 0.9$ and $\rho_{in} = 10^3$ the initial temperature is $T_{in} \approx m$. Such a large rise of temperature is explained by much larger energy release into plasma by $\chi\bar{\chi}$ annihilation, which is absent in the case of $|\kappa| = 1$ when antiparticles of χ are initially absent.

We have numerically integrated this equation with constraint (30). The solution is divided into two parts: initially with non-zero and running $C(a)$ (condensate), $\mu = m$,

and running $T(a)$. Then at some stage C reaches zero and the condensate disappears. After that the solution is searched for $C \equiv 0$ but running $\mu(a)$ and $T(a)$. The details are presented in the Appendix. The calculations are very much simplified in the limit of low temperatures when $T \ll m$. The functions ρ_{tot} and p_{tot} in this limit are simple algebraic functions and not integrals over energy and the evolution equation turns into:

$$\frac{\xi'}{\xi} \frac{2\alpha_2 \xi^{-5/2} + \beta_1 \xi^{-2} + 4\gamma_3 \xi^{-4}}{4\alpha_2 \xi^{-5/2} + 3\beta_1 \xi^{-2} + 4\gamma_3 \xi^{-4}} = \frac{1}{a} \tag{34}$$

Depending upon which term dominates in the ξ dependent coefficients in the l.h.s. of this equation we obtain different regimes of the cosmological cooling. At relatively small ξ, when ξ^{-4} dominates, the temperature drops as $T \sim 1/a$. It is the usual law of relativistic cooling. If the first term, $\xi^{-5/2}$, dominates, $T \sim 1/a^2$. This is also the known regime for dominating non-relativistic matter. However, if the term ξ^{-2} is dominant, the temperature drops unusually fast:

$$T \sim 1/a^3 \tag{35}$$

We have found that this regime is the typical one in the course of the condensate evaporation, so the temperature becomes very low at the end. It is quite unusual situation. Normally one would expect that a source of energy created by the evaporating condensate would lead to plasma heating and a slower decrease of the temperature in the course of expansion. We observe an opposite picture. Probably this phenomenon is induced by a very slow decay of the condensate and efficient cooling of the particles in the plasma by the scattering on the "refrigerator" created by a large number of cold χ-particles at rest.

Here we would like to remark that the derivation of the temperature's evolution law can be extended to the case of a different particle content, even if eq. (17) is no longer true, as we have already pointed out. In fact in such a system, when both energy and charge densities are dominated by ξ^{-2} terms, we can generalize eq. (35) to the following:

$$T \sim 1/a^{3z} \tag{36}$$

where:

$$z = \frac{2N_l \mu_l^2 + 2N_f \mu_f^2 - N_l \mu_l}{3N_l \mu_l^2 + 3N_f \mu_f^2 - 2N_l \mu_l} \tag{37}$$

If we now insert the chemical potentials given by the equations of equilibrium with respect to reaction (13), together with the equality of charges $Q_B - Q_L = 0$, we will find for a different particle contents, that $z = 1$, which brings back eq. (35).

In non-relativistic case, as well as in the relativistic one, with dominating condensate, the behavior of $C(a)$ can be predicted analytically from evolution equations (29) and (31):

$$C' = -\frac{3}{a}(C - u(T)) \tag{38}$$

where $u(T)$ contains the temperature-dependent terms which contribute to the energy or to the charge density. Thus if the condensate dominates the energy and charge densities, i.e. $C \gg u(T)$, the evaporation is described by the law $C \sim 1/a^3$.

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The evolution of the interesting quantities is presented in figures 6 and 7. In accordance with the arguments presented above the amplitude of the condensate follows the simple law:

$$C \sim 1/a^3 \tag{39}$$

which holds through most of its life. Only when the energy density of the condensate becomes approximately equal to that of the plasma, $C(t)$ starts to drop faster and at some moment it reaches zero. All the plots with $\mu = m = const$ have been stopped at the precise moment of the complete evaporation.

If the initial temperature is high, we do not have an accurate analytical approximation for the energy, pressure, and charge density of χ with the large chemical potential $\mu = m$. In this case we need to solve differential equation (32) with the integrals for ρ, p, and Q taken numerically. The solutions are presented in fig. 8. Initially the temperature drops as $T \sim 1/a$ and later, when it becomes smaller than m before the complete evaporation of the condensate, the regime changes to that discussed above in the non-relativistic case.

5   Results and discussion

The obtained results are presented in figs. 6, 7, 8, and 9. All the quantities which appears in these figures are normalized to a proper power of the mass of χ, namely:

$$T \to T/m, \quad \mu \to \mu/m,$$
$$Q \to Q/m^3, \quad \rho \to \rho/m^4$$

and they are expressed as functions of the scale factor ratio a/a_{in}. Most of these graphs are in the log-log scale, except for the 'Power of T' and 'Zoom of C' plots, which are in the log-linear scale.

In fig. 6 a special case $\kappa = 1$ is presented. We see in the upper left box how the temperature evolves, while the box below (middle left), named 'Power of T', shows that the cooling follows the law $T \sim 1/a^3$. The same law holds for the condensate, as it is shown in the right upper box, that is, $C \sim 1/a^3$, while the middle right box called 'Zoom of C', is a zoom of its final evolution, and shows the instant of the complete evaporation. The calculated cosmological charge asymmetry, $\beta = Q/n_\gamma = Q/0.24\,T^3$, is presented in the left lower box: there are lines for the condensate (solid line), decay products (dashed line), and relativistic light charged particles (dot-dashed line). It can be seen that the produced asymmetry strongly rises during the non-relativistic regime. This is understandable because the temperature drops very fast, $T^3 \sim a^{-9}$ while the charge density drops only as $Q \sim a^{-3}$. We have found that the ratio of $\beta(a = 1)$ to β at $C = 0$ is, in this case, of the order of 10^8. The last box (lower right) shows how entropy in comoving volume, $S = sa^3 = a^3(\rho + p)/T$, evolves: we see, as it was for β, that the fast cooling results in S rising as a^3 because the main contribution to the energy density comes from the condensate, which drops only as $1/a^3$.

In fig. 7 the same graphs are presented, but for different initial values of temperature and condensate which correspond to $\kappa = 0.9$, and respectively to $T_{in} \approx 0.65m$. We see

that all the quantities evolve similarly to the previous case presented in figure 6, but here one can better see how the behavior of temperature switches from $T \sim 1/a$ to $T \sim 1/a^3$ with growing scale factor. The moment when it happens is precisely the moment when the ξ^{-2}-terms in the energy and charge densities start to dominate over the standard ξ^{-4} ones, see eq. (34). In this situation we have found the ratio of the final to initial charge density to be about 10^3.

Next, in fig. 8 the case of initial relativistic state is presented. In order to have relativistic initial state we had to choose $\kappa = 0.1$ and a huge initial total energy density, $\rho_{in} \approx 10^6 m^4$. An important thing to be noticed here is that the condensate can evaporate completely only when the non-relativistic limit is reached, i.e. $T < m$.

Finally, in fig. 9 we present evolution of temperature, chemical potential, energy densities, baryonic charge, and entropy, when the condensate has completely evaporated. In the first box, clockwise from upper left corner, we see that the temperature follows the standard evolution law for a relativistic plasma, $T \sim 1/a$. In the second box we have the same graph for the chemical potential, which also behaves as $\mu \sim 1/a$. The third box contains: entropy in comoving volume (thick solid line); charge density, or to be more precise β, contributed by relativistic (thin solid line) and non-relativistic (thin dashed line[1]) species. Finally, the fourth box (lower left corner) shows the evolution and the ratios of the energy densities of charged relativistic particles (solid line), photons (dot-dashed line), and non-relativistic χ-particles (dashed line). We see that both entropy in comoving volume and β are conserved. One may wonder if a non-zero chemical potential could break the entropy conservation law, but it can be easily checked that this is not so because every term in the expression of charge density and entropy in comoving volume behave as $1/a^3$. This, together with the fact that the energy density of non-relativistic particles (as well as their charge density) is orders of magnitude smaller than that of the relativistic ones, as one can see in the 4th box, gives the result just obtained. Hence the Affleck and Dine scenario of baryogenesis could indeed lead to a huge baryon asymmetry of the universe.

To dilute the asymmetry to the observed level a subsequent release of entropy is necessary or evaporation into initially hot plasma. Both such mechanisms were discussed recently in ref. [4], where one can find references to many earlier papers.

A   Appendix

In this appendix we present some details of the numerical solution of the differential equations[2]:

$$\begin{cases} \dot{Q}_{tot} = -3HQ_{tot} \\ \dot{\rho}_{tot} = -3H\left(\rho_{tot} + p_{tot}\right) \end{cases} \Rightarrow \begin{cases} Q'_{tot} = -3\,Q_{tot}/a \\ \rho'_{tot} = -3\left(\rho_{tot} + p_{tot}\right)/a \end{cases} \tag{40}$$

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[1]This line, as well as the thin dashed one in the fourth box, seems to be vertical but in fact it is not: it is an extremely fast decrease of the contribution of non-relativistic χ and $\bar{\chi}$ particles.

[2]All the quantities here are normalized to proper powers of χ-mass, e.g. $T \to T/m$, $\rho \to \rho/m^4$.

We have to insert into these expressions the explicit values of Q_{tot} and ρ_{tot}, which depend either upon $T(a)$ and $C(a)$ when $\mu = m$, or upon $T(a)$ and $\mu(a)$ when $C = 0$, that is:

$$\begin{cases} Q_{tot} = Q_{tot}(T, C) \\ \rho_{tot} = \rho_{tot}(T, C) \end{cases} \quad \text{or} \quad \begin{cases} Q_{tot} = Q_{tot}(T, \mu) \\ \rho_{tot} = \rho_{tot}(T, \mu) \end{cases} \tag{41}$$

In the first case, when $C \neq 0$, these equations read:

$$\begin{cases} Q'_{tot} = C' + \zeta\, T' \\ \rho'_{tot} = C' + \eta\, T' \end{cases} \tag{42}$$

where $\zeta(T)$ and $\eta(T)$ are the derivatives of $Q_{tot}(T, C)$ and $\rho_{tot}(T, C)$ with respect to T. The system can be solved together with equations (40) to have:

$$\begin{cases} (\eta - \zeta)\, T' = 3\left[Q_{tot} - (\rho_{tot} + p_{tot})\right]/a \\[2ex] (\eta - \zeta)\, C' = 3\left[\zeta(\rho_{tot} + p_{tot}) - \eta\, Q_{tot}\right]/a \end{cases} \tag{43}$$

As for the second case, when $C = 0$, the derivatives of Q_{tot} and ρ_{tot} are:

$$\begin{cases} Q'_{tot} = f\, \mu' + g\, T' \\ \rho'_{tot} = h\, \mu' + l\, T' \end{cases} \tag{44}$$

where $f(\mu, T)$ and $h(\mu, T)$ are the derivatives of $Q_{tot}(T, \mu)$ and $\rho_{tot}(T, \mu)$ with respect to μ, while $g(\mu, T)$ and $f(\mu, T)$ are the derivatives of $Q_{tot}(T, \mu)$ and $\rho_{tot}(T, \mu)$ with respect to T. Again, the system can be solved together with equations (40) to have:

$$\begin{cases} (gh - fl)\, T' = 3\left[f(\rho_{tot} + p_{tot}) - h\, Q_{tot}\right]/a \\[2ex] f(gh - fl)\, \mu' = 3\left[fl\, Q_{tot} - gf(\rho_{tot} + p_{tot})\right]/a \end{cases} \tag{45}$$

Now the system of differential equations (45), as well as system (43), can be solved numerically.

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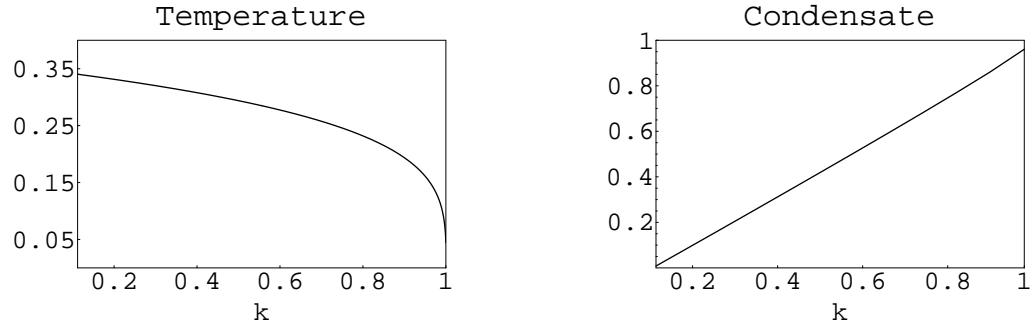
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Figure 1: Initial values of temperature and condensate as functions of κ, with the fixed value of the total energy density $m^{-4}\rho_{tot} = 1$.
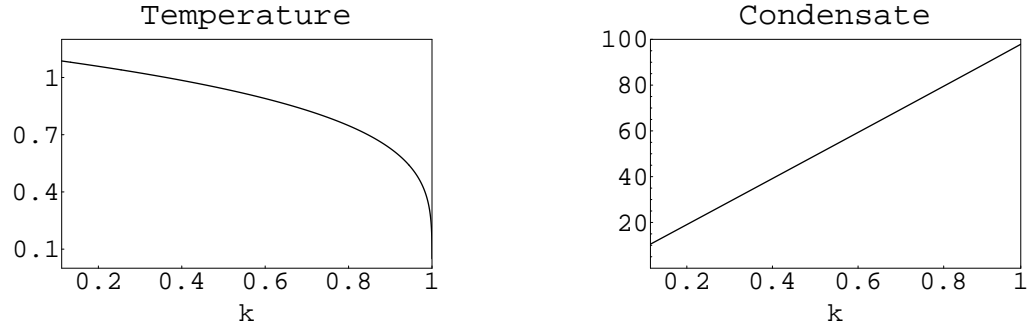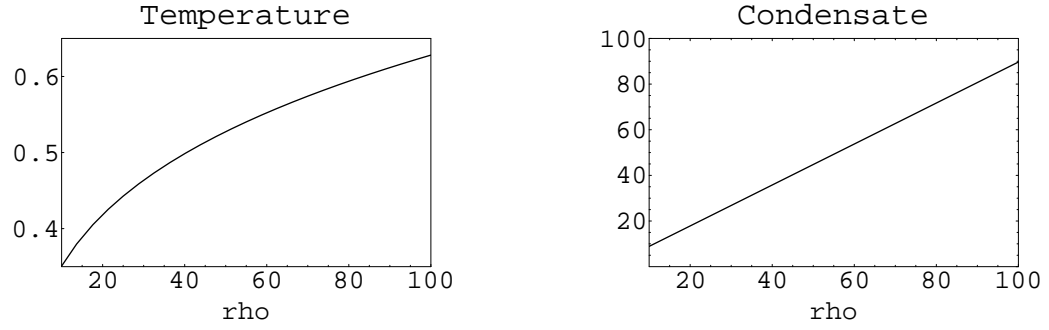

Figure 2: Initial values of temperature and condensate as functions of κ, with the fixed value of the total energy density $m^{-4}\rho_{tot} = 100$.
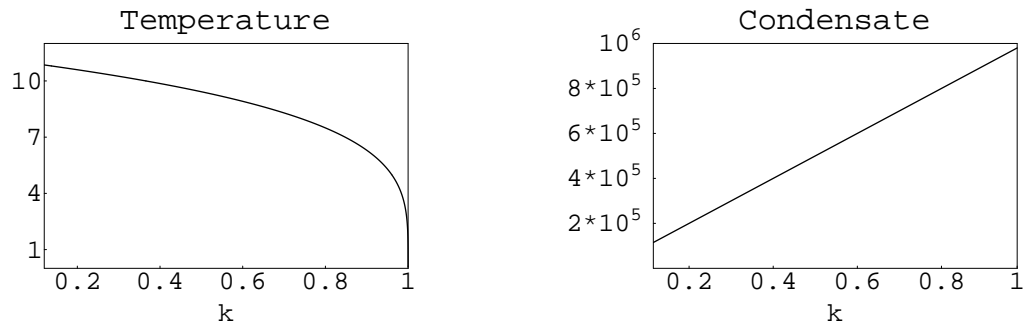
Figure 3: Initial values of temperature and condensate as functions of κ, with the fixed value of the total energy density $m^{-4}\rho_{tot} = 10^6$.

Figure 4: Initial values of temperature and condensate as functions of the energy density, with fixed $\kappa = 0.9$.
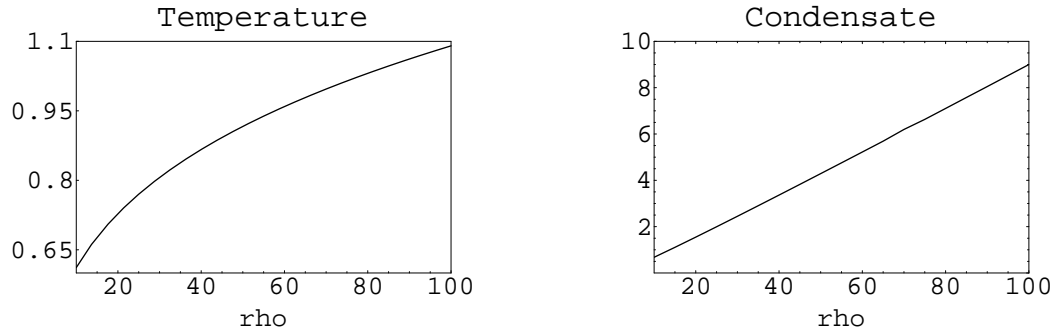
Figure 5: Initial values of temperature and condensate as functions of the energy density, with fixed $\kappa = 0.1$.
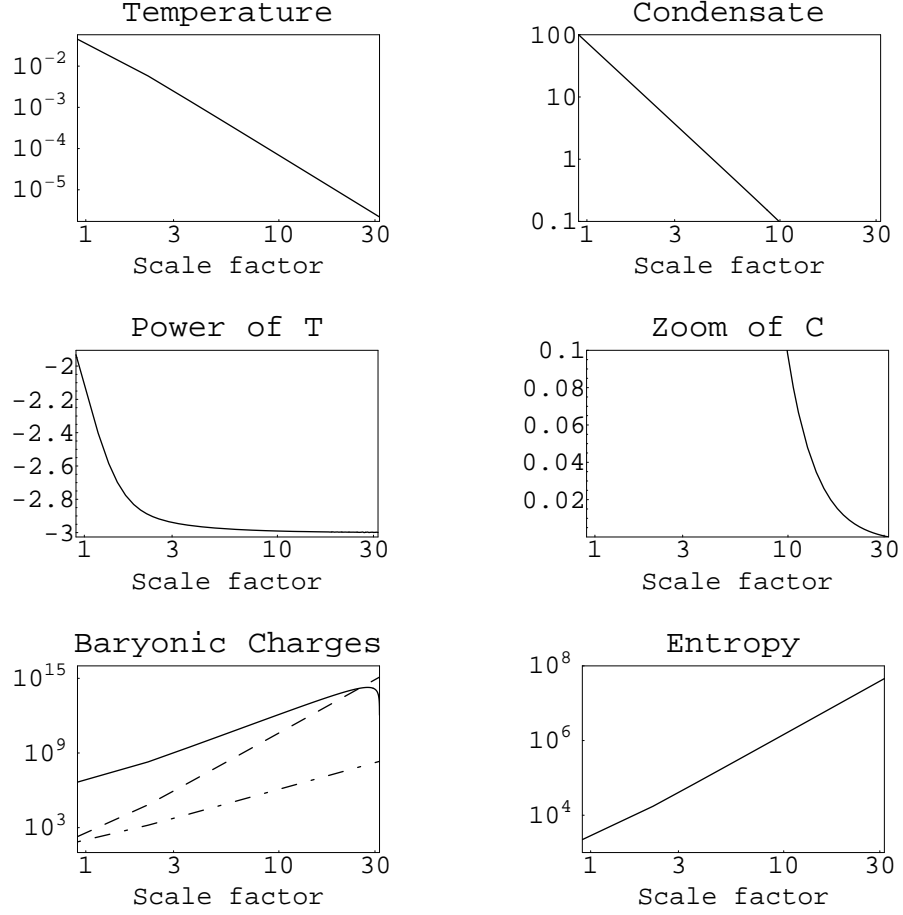
$C_{in} \sim 100$

Temperature

Condensate

Power of T

Zoom of C

Baryonic Charges

Entropy

Figure 6: This figure refers to the special case $\kappa = 1$. Clockwise from upper left corner: evolution of temperature, evolution of the condensate (upper right), the complete evaporation of the condensate (middle right), evolution of entropy in comoving volume (lower right), evolution of baryonic charges (lower left corner), and the power of $T(a)$ (middle left), as functions of the scale factor a. The 'Baryonic Charges' graph has the following lines: solid line is for the condensate; dashed line is for χ and $\bar{\chi}$ particles; dot-dashed line is for relativistic charged particles.
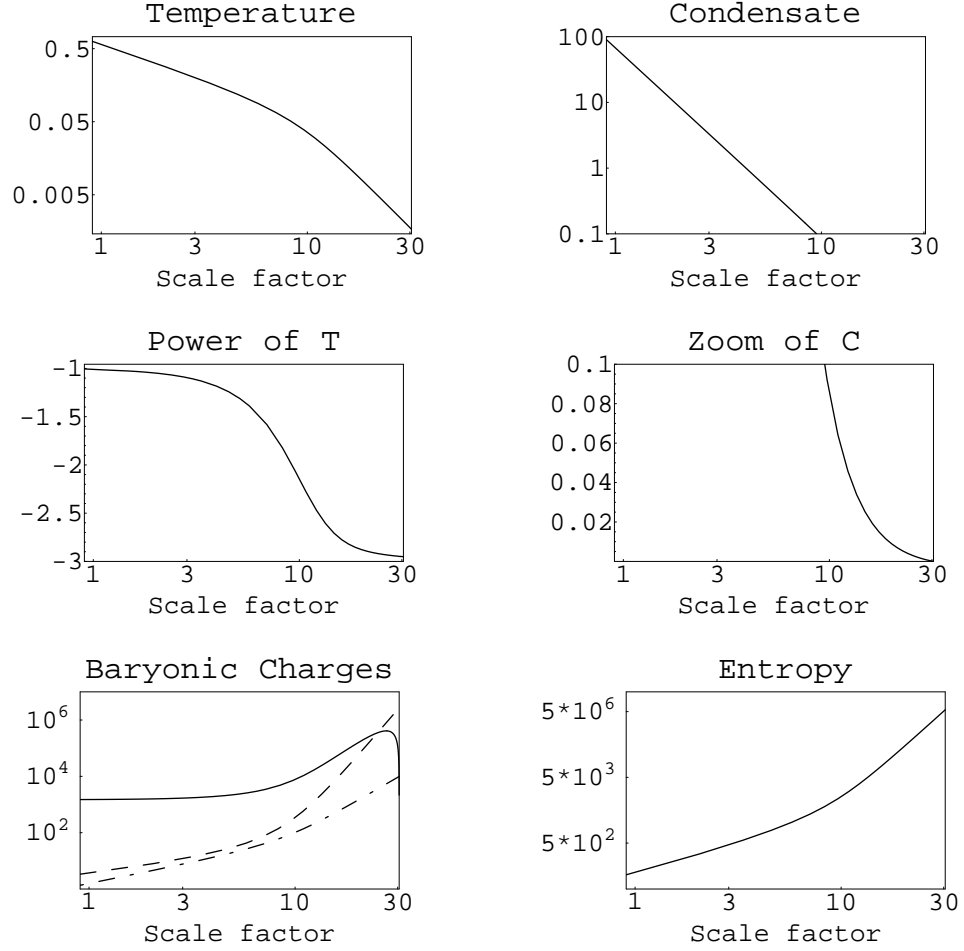
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Figure 7: This figure refers to the case $\kappa = 0.9$, while $m^{-4}\rho_{tot} = 100$. Clockwise from upper left corner: evolution of temperature, evolution of the condensate (upper right), the complete evaporation of the condensate (middle right), evolution of entropy in comoving volume (lower right), evolution of baryonic charges (lower left corner), and the power of $T(a)$ (middle left), as functions of the scale factor a. The 'Baryonic Charges' graph has the following lines: solid line is for the condensate; dashed line is for χ and $\bar{\chi}$ particles; dot-dashed line is for relativistic charged particles.
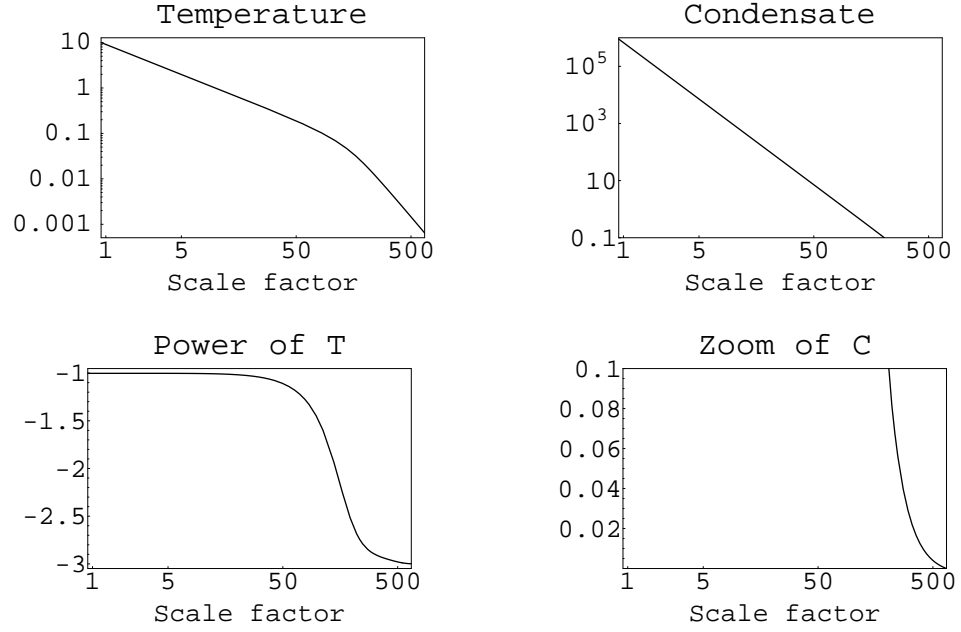
Figure 8: This figure refers to the relativistic initial state, that is $T > m$. Clockwise from upper left corner: evolution of temperature, evolution of the condensate, the complete evaporation of the condensate, the power of $T(a)$, as functions of the scale factor a. We see that the evaporation happens only when temperature has dropped below m.
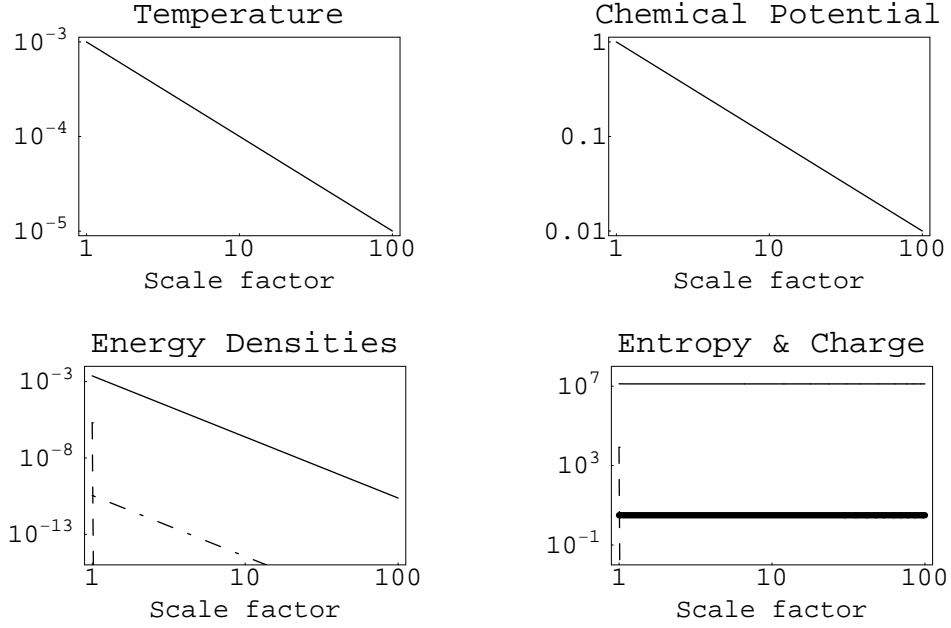
Figure 9: This figure refers to the second part of the problem, when the condensate has completely evaporated and μ is no longer a constant. Clockwise from upper left corner: evolution of temperature, chemical potential, entropy in comoving volume and baryonic charges, and energy densities, as functions of the scale factor a. The 'Energy densities' graph has the following lines: solid line is for relativistic charged particles; dashed line is for photons; dot-dashed line is for χ and $\bar{\chi}$ particles. The 'Entropy & Charge' graph has the following lines: the solid line is the charge of relativistic particles; the dashed line is the charge of χ and $\bar{\chi}$ particles, and the thick solid line is the entropy.